# Written Exam at the Department of Economics summer 2019 

# Microeconomics III 

Final Exam

June 24, 2019
(2-hour closed book exam)

Answers only in English.

This exam question consists of 3 pages in total (including the current page)

## Falling ill during the exam

If you fall ill during an examination at Peter Bangs Vej, you must:

- contact an invigilator who will show you how to register and submit a blank exam paper.
- leave the examination.
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five
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## Be careful not to cheat at exams!

You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam


## PLEASE ANSWER ALL QUESTIONS PLEASE EXPLAIN YOUR ANSWERS

1. Consider the following game $G$, where Player 1 chooses the row and Player 2 simultaneously chooses the column.

$$
\text { Player } 2
$$

Player 1

|  | $D$ |  |  |
| :---: | :---: | :---: | :---: |
| $A$ | $F$ |  |  |
|  | $8,-1$ | 2,0 | 1,0 |
|  | 7,6 | 0,1 | 0,3 |
| $C$ | 2,2 | 4,3 | 0,0 |
|  |  |  |  |

(a) Show which strategies in $G$ are eliminated by following the procedure of 'Iterated Elimination of Strictly Dominated Strategies'.
(b) Find all Nash equilibria (NE), pure and mixed, in $G$. Show which NE gives the highest payoff to both players, and denote this equilibrium strategy profile by $e(1)$.
(c) Now consider the game $G(2)$, which consists of the stage game $G$ repeated two times. For the time being, you can assume that players have discount factor $\delta=1$, i.e. they place equal weight on period-1 and period-2 payoffs.

Find one pure strategy Subgame Perfect Nash Equilibrium (SPNE) where both players are better off than they were in part (b). That is, where both players earn at least as much as in $e(1)$, in each period, and earn strictly more than in $e(1)$ in at least one period. (NOTE: make sure to consider deviations in any subgame). Denote the equilibrium strategy profile you found by $e(2)$.
(d) Now consider the game $G(\infty)$, which consists of the stage game $G$ repeated infinitely many times. Assume that players discount future payoffs with factor $\delta \geq 1 / 7$.

Find one pure strategy SPNE where both players are better off than they were in part (c). That is, where both players earn at least as much as in $e(2)$, in each period, and earn strictly more than in $e(2)$ in at least one period.
2. Suppose we are in a private value auction setting. There are two bidders, $i=1,2$. They have valuation $v_{1}$ and $v_{2}$, respectively. These values are distributed independently and uniformly with $v \sim U(3,4)$. The auction format is sealed-bid first price. In case of a tie, a fair coin is flipped to determine the winner.
(a) Suppose player j uses the strategy $b\left(v_{j}\right)=c v_{j}+d$, where c and d are constants. Show that if bidder $i \neq j$ bids $b_{i}$, his probability of winning is

$$
P\left(i \text { wins } \mid b_{i}\right)=\frac{b_{i}-d-3 c}{c}
$$

whenever $3 c+d \leq b_{i} \leq 4 c+d$.
Hint: Recall that if $x \sim U(a, b)$ then $P(x \leq y)=\frac{y-a}{b-a}$
(b) Using the result in (a), show that there is a symmetric Bayes Nash equilibrium (BNE) in linear strategies $b\left(v_{i}\right)=c v_{i}+d, i=1,2$. Find c and d.
(c) Check that, in the symmetric BNE from part (b), the lowest-type bidder will place a bid equal to his valuation, whereas all other bidders will bid strictly less than their valuation, and comment briefly on why this is the case. Does this relate to the 'winner's curse'? Briefly justify your answer (NOTE: please attempt this subquestion even if you did not complete part (b)).
3. Now consider the following game $G^{\prime}$ :


Note that in this game, the prior probability that the sender is of type 1 is equal to 0.1 .
(a) Briefly explain whether $G^{\prime}$ is a static or a dynamic game (1 sentence), and whether or not $G^{\prime}$ is a cheap talk game ( 1 sentence).
(b) Find a separating equilibrium in $G^{\prime}$, and find a pooling equilibrium where both sender types play $R$.
(c) Check whether the two equilibria you found in part (a) satisfy Signaling Requirements 5 ('strict domination') and 6 ('equilibrium domination').
(d) Now suppose that we modify $G^{\prime}$, so that the prior probability of the sender being type $t_{1}$ is given by the parameter $\alpha \in[0,1]$. For what values of $\alpha$ does a pooling equilibrium exist where both sender types play $R$ ? (NOTE: parts (a) and (b) of this question considered the special case where $\alpha=0.1$ ).
(e) Briefly comment on why a pooling equilibrium where both sender types play $R$ exists for some values of $\alpha$, but not for others. What is the intuition for this result?

